

Varying and inverting the mass hierarchy in collisional energy loss

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Heavy ion collisions at RHIC and at the LHC give access to the medium-induced suppression patterns of heavy-flavored single inclusive hadron spectra at high transverse momentum. This opens novel opportunities for a detailed characterization of the medium produced in the collision. In this note, we point out that the capacity of a QCD medium to absorb the recoil of a partonic projectile is an independent signature, which may differ for different media at the same density. In particular, while the mass hierarchy (i.e., the projectile mass dependence) of radiative energy loss depends solely on a property of the projectile, the mass hierarchy of collisional energy loss depends significantly on properties of the medium. By varying these properties in a class of models, we find that the mass hierarchy of collisional parton energy loss can be modified considerably and can even be inverted, compared to that of radiative parton energy loss. This may help to disentangle the relative strengths of radiative and collisional contributions to jet quenching, and it may be employed to constrain properties of the produced QCD medium beyond its density.

1. Introduction

'Jet quenching', the energy loss of high momentum partons in hot and dense QCD matter is at the basis of the strong medium-induced suppression of hadronic high- p_T spectra, discovered at RHIC [1]. It is also expected to dominate the physics at high transverse momenta in heavy ion collisions at the LHC [2]. The strong sensitivity of high- p_T hadronic spectra to the nuclear environment makes jet quenching a promising tool for the characterization of properties of the matter produced in heavy ion collisions. However, the accuracy of this tool depends largely on understanding the microscopic mechanism by which the medium affects the stopping and fragmentation of partonic projectiles. At sufficiently high projectile energy, an essentially recoilless radiative energy loss mechanism is expected to dominate on general kinematic grounds [3–7]. At sufficiently small projectile energies, however, recoil is expected to be non-negligible. Several recent model studies [8–12] attribute a sizable role to collisional mechanisms mediated via elastic interactions. But an experimental strategy to disentangle the effects of elastic and inelastic interactions (a.k.a. collisional and radiative energy loss) is missing so far. This makes it interesting to look for experimental signatures which are qualitatively different for both mechanisms.

For the radiative energy loss mechanism, we know that the dependence of parton energy loss on the mass and color charge of the partonic projectile shows a characteristic hierarchy. Due

to their larger color charge, gluons radiate more than light quarks. And light quarks radiate more than heavy quarks, since radiation is suppressed as a function of projectile mass [13–17]. Remarkably, this hierarchy of radiative parton energy loss is determined solely by properties of the partonic projectile, namely its color charge and its mass. Properties of the medium affect the absolute strength of medium modifications, but leave its relative dependence on parton identity unchanged. The current understanding of the characteristic tell-tale signs of collisional energy loss is less complete. To the best of our knowledge, one has not yet addressed the question to what extent the mass hierarchy of collisional energy loss depends on properties of the medium or on properties of the projectile.

To discuss medium modifications of parton propagation, one must specify the properties of the medium. If the medium is in thermal equilibrium, then all properties of the medium depend on temperature only. In principle, this fixes the relative strength of collisional and radiative energy loss. In practice, however, this relative strength may be difficult to evaluate for the temperature range which is in reach of heavy ion collisions. In addition, the systems produced in heavy ion collisions may show interesting deviations from the idealization of a thermal heat bath. For these reasons, we do not presuppose in the following a unique relation between the density of the medium and other features (such as the masses of quasi-particles or the capacity to absorb recoil). Rather, we shall vary these features independently within some parameter range with the view of determining them finally in a comparison to data.

In section 2 of this paper, we introduce a class of models of the medium, and we discuss how these models differ in their capacity of absorbing recoil. In section 3 and 4, we then point out that depending on the model-dependent capacity of the medium to absorb recoil, the strength of collisional energy loss and its dependence on projectile mass can vary strongly. We finally comment on the implications of our findings.

2. The model

Several calculations of radiative energy loss model the medium in terms of a set of static colored scattering centers [3–6]. Since medium-induced radiation is expected to depend mainly on the transverse color field strength presented by the medium to the projectile, and since a set of static scattering centers parametrizes conveniently this color field strength, such a simple model captures the main feature relevant for radiative energy loss. In the same spirit, many recent calculations of collisional energy loss model the medium as either a set of massless particles with thermal momentum distribution [8, 9], or as a set of initially static massive scattering centers [10]. By making the target scattering centers dynamical, these models parametrize not only the color field strength but also the capacity of the medium to absorb recoil.

Following this approach, we consider models, which are characterized by two parameters: the mass m_t of colored scattering centers in the fundamental (q) and adjoint (g) representation, and a parameter T which characterizes the momentum distribution of these scattering centers,

$$n_q(k) = f_{\text{scale}} \frac{1}{(2\pi)^3} \frac{12n_f}{e^{E_k/T} + 1}, \quad n_g(k) = f_{\text{scale}} \frac{1}{(2\pi)^3} \frac{16}{e^{E_k/T} - 1}, \quad (2.1)$$

where $E_k = \sqrt{m_t^2 + k^2}$ and $n_f = 2$. The mass m_t does not depend on the temperature T . If the scale factor is $f_{\text{scale}} = 1$, then the number density of scattering centers decreases with increasing m_t . To disentangle observable consequences of a decreasing density from other target mass dependent effects, we also consider the case

$$f_{\text{scale}}(m_t) = \frac{\int d^3k (n_q(k) + \frac{9}{4}n_g(k))|_{m_t=200 \text{ MeV}}}{\int d^3k (n_q(k) + \frac{9}{4}n_g(k))|_{m_t}}. \quad (2.2)$$

With this normalization, the integrated effective density of scattering centers entering parton energy loss (see eq. (2.3) below) does not depend on m_t .

We consider a light or heavy projectile quark Q , which propagates through the class of model targets described above. This quark accumulates collisional momentum loss Δp_Q per unit path length dx by incoherent elastic scattering on target partons,

$$\frac{d\Delta p_Q}{dx} = \frac{1}{v_Q} \int dp_f (p - p_f) \int k^2 dk (n_q(k) + \frac{9}{4}n_g(k)) \frac{d\sigma_{Qq}^{\text{int}}(k, p_f)}{dp_f}. \quad (2.3)$$

Here, p is the initial and p_f the final momentum of the projectile Q , and v_Q denotes its velocity in the rest frame of the medium. We approximate the elastic Qg scattering cross section by the leading t -channel exchange, such that $\frac{d\sigma_{Qg}^{\text{int}}(\langle k \rangle, p_f)}{dp_f} = \frac{C_A}{C_F} \frac{d\sigma_{Qq}^{\text{int}}(\langle k \rangle, p_f)}{dp_f}$. The elastic scattering cross section is of the general form

$$\frac{d\sigma^{\text{int}}}{dp_f} = 2\pi \int d(\cos \psi) \frac{1}{4p^0 k^0} |\mathcal{M}|^2 d\Phi, \quad (2.4)$$

where $2\pi \int d(\cos \psi)$ denotes the integration over the direction of the incoming target particle, and $d\Phi$ denotes the phase space volume. The model is fully defined once the scattering matrix is specified.

For the elastic scattering matrix element \mathcal{M} , we use the expression to lowest order in α_s with single gluon exchange in the t -channel described by the HTL-resummed propagator [18, 19]. This is the starting point of many works on collisional energy loss [20, 21]. As in the recent work of Djordjevic [8], we compute the matrix element for Q - q scattering without any assumption on the smallness of masses or energy transfers. In summary, our calculation is a standard calculation of collisional energy loss in which t -channel exchanges are regulated by thermal propagators, but in which - in contrast to previous models - the mass of the scattering centers in the medium is treated as an independent parameter m_t . Also, in contrast to the constant coupling constant $\alpha_s = 0.3$ used in [8, 9], we use a running coupling constant $\alpha_s(\mu_D^2 + k_T^2)$,

where $\mu_D^2 = g(\mu_D^2)^2 T^2 (1 + n_f/6)$ defines the Debye screening mass [22]. For $T = 225$ MeV, this choice corresponds to $\mu_D = 680$ MeV.

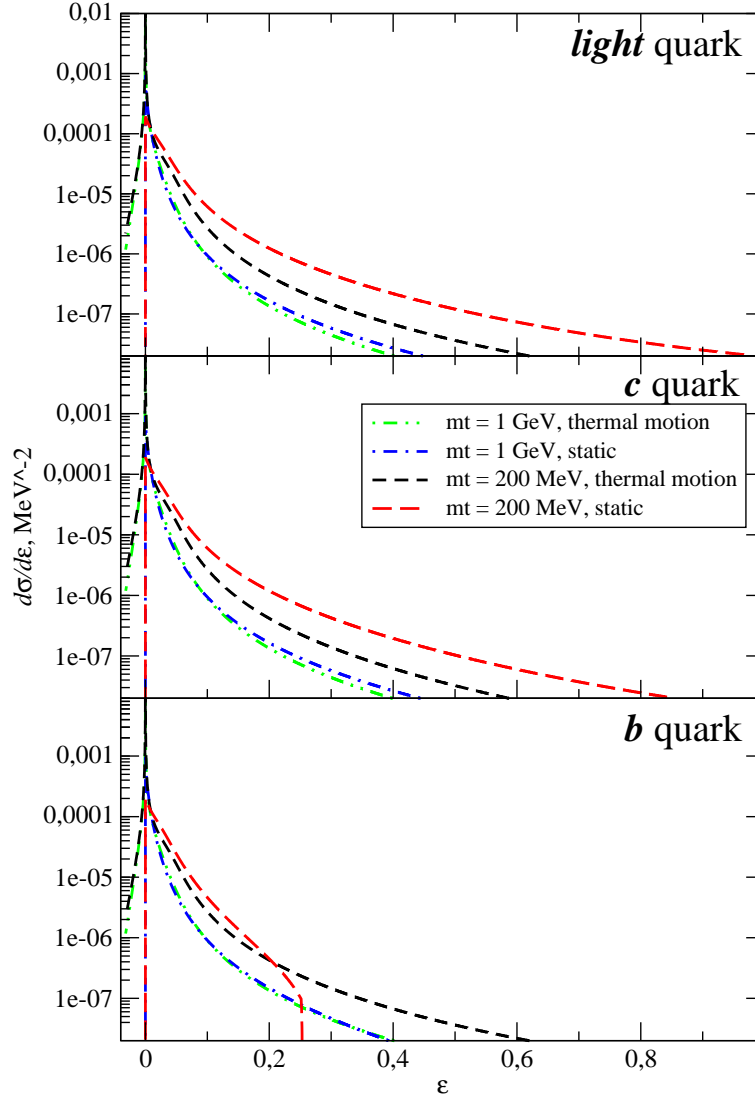


Fig. 1. The cross section of fractional energy loss $\epsilon = \Delta E/E$ in one collision, calculated from (2.4) for different partonic projectiles and for different masses m_t of the scattering centers in the medium. The initial projectile momentum is $p = 20$ GeV. The target scattering centers are either initially at rest ('static'), or they show a thermal momentum distribution with $T = 225$ MeV ('thermal motion'). In the latter case, the cross section is averaged over all possible target particle momenta.

3. Fractional collisional energy loss

In Fig. 1, we plot the cross section for the fractional energy loss $\epsilon = \Delta E/E$, suffered by a relativistic $p = 20$ GeV partonic projectile in a single collision. As partonic projectiles, we consider light quarks ($m_q = 200$ MeV), charm quarks ($m_c = 1200$ MeV) and bottom quarks

($m_b = 4750$ MeV). The scattering centers in the medium have a target mass m_t . In general, one sees from Fig. 1 that collisional energy loss peaks always at relatively small momentum fractions. This comes from the fact, that elastic interactions are dominated by small-angle scattering, in which longitudinal momentum transfer is small.

Fig. 1 compares the fractional energy loss for a model, in which the scattering centers in the target are initially at rest ('static'), to a model in which their distribution is thermal with $T = 225$ MeV. In heavy ion collisions, one expects that the scattering centers in the medium show some random (thermal) motion, so one favors a thermal distribution on physical grounds. But static distributions have been considered recently [10] for scattering centers with target masses $m_t = 200$ MeV. Our results for this case agree with those of Ref. [10]. This illustrates that our set-up is consistent with standard collisional energy loss calculation. An increase of the target mass from $m_t = 200$ MeV to $m_t = 1$ GeV in this static case leads generally to a significant reduction of collisional energy loss. In this sense, the capacity of this model of the medium to absorb recoil can be varied by varying m_t . This is, of course, expected on general kinematic grounds. In the limit of infinitely massive scattering centers in the target, the medium would not absorb any recoil. The collisional interactions would not be visible in an energy degradation of the projectile, but only in its momentum broadening.

The maximal possible value of momentum transfer in a single collision is restricted by the available phase space. This restriction is especially pronounced for an initially static target. For instance, a heavy projectile, colliding with a light static target cannot lose but a certain fraction of its total energy. As is seen in the lower panel of Fig. 1, this leads for heavy b -quark projectiles to a severe restriction of energy loss for light target mass. The restriction is weakened if the target particle is heavier or if it carries some initial randomly distributed momentum.

In comparing the models of scattering centers with thermal and with static momentum distribution, shown in Fig. 1, we note that collisional energy loss for a relativistic projectile appears to depend mainly on the average total energy of the scattering centers in the target, rather than on their mass. Since a light particle in contact with a heat bath of temperature $T = 225$ MeV has an average kinetic momentum of $\gtrsim 600$ MeV, this explains qualitatively the significant differences between both models for a small target mass $m_t = 200$ MeV, where the average total energy of scattering centers exceeds their rest mass by far. The same argument accounts for the much less pronounced differences for $m_t = 1$ GeV. It may also explain why the case of small target mass $m_t = 200$ MeV with thermal motion lies in between the curves for static scenarios with $m_t = 200$ MeV and $m_t = 1$ GeV, respectively.

A thermal distribution of scattering centers leads also to some novel features in the cross section of fractional energy loss. In particular, the cross sections in Fig. 1 show non-vanishing contributions for negative ϵ , since a projectile may (though with small probability) gain energy in scattering with a target component. With increasing target mass, the probability of losing a significant fraction of the total initial energy decreases for the same generic kinematic reasons as in the case of the static scenario, described above. So, also in this case varying the target mass changes the capacity of the medium to absorb recoil.

4. Mass Hierarchy of Collisional Energy Loss

Fig. 2 shows results for the average energy loss of a partonic projectile of momentum p , which propagates through $L = 5$ fm of matter. The medium is characterized by scattering centers of target mass m_t . We consider *i*) the case for which the massive target particles show a thermal distribution [eq. (2.1) with $f_{\text{scale}} = 1$] and *ii*) the case that the momentum distribution of target particles follows case *i*) but that the total density is fixed by eq. (2.2) to an m_t -independent value. For projectile particles whose momentum does not differ much from typical momenta in the media, the distinction between projectile and target becomes questionable. For this reason, we show results for projectile momenta of $p > 2$ GeV only, though our model would smoothly extend to softer momenta.

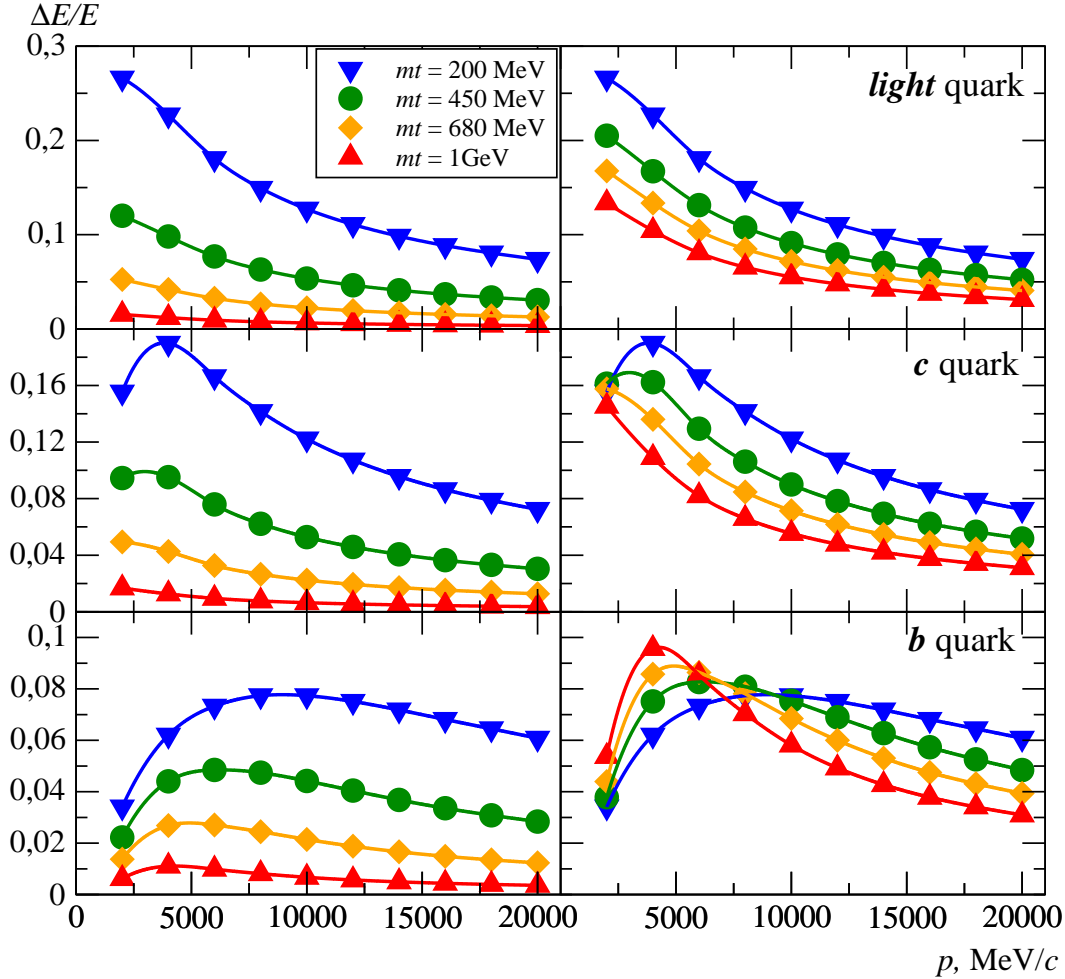


Fig. 2. Collisional parton energy loss fraction $\Delta E/E$ as a function of projectile momentum p calculated from equation (2.3) for light quarks ($m_q = 200$ MeV, upper panel), charm quarks ($m_c = 1.2$ GeV, middle panel) and bottom quarks ($m_b = 4.75$ GeV, lower panel). The path length $L = 5$ fm, other parameters are chosen as for Fig. 1. The massive target particles in the medium are distributed according to (2.1) with $f_{\text{scale}} = 1$ (left column) or f_{scale} defined by eq. (2.2) (right column).

We consider first the case *i*) that the massive target particles follow the thermal distribution (2.1) with $f_{\text{scale}} = 1$. In this case, the average collisional energy loss drops strongly with increasing target mass for all values of projectile momentum, see left hand side of Fig. 2. For $m_t = 1$ GeV, the average energy loss is a factor of order 10 smaller than for $m_t = 200$ MeV. This strong dependence on target mass m_t has two different origins. First, by increasing m_t , the density of scattering centers decreases and this leads to a strong decrease of ΔE . Second, changing m_t also changes the average energy loss per collision.

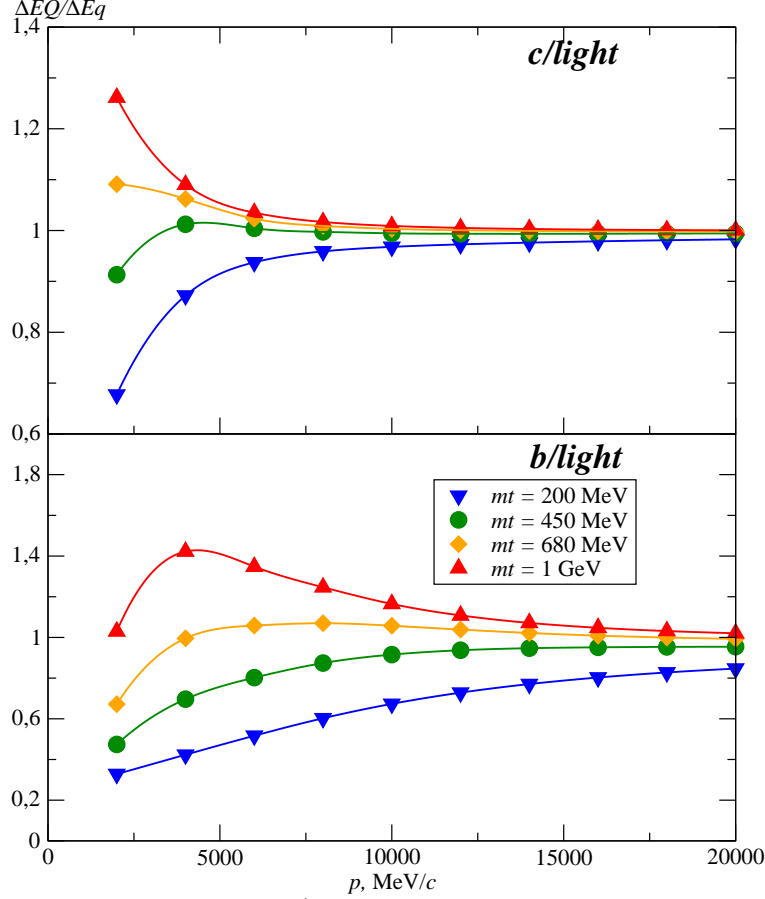


Fig. 3. The heavy-to-light ratio $\Delta E_Q/\Delta E_q$ of collisional energy loss for charm quarks (upper panel) and bottom quarks (lower panel), compared to that of light quarks ($m_q = 200$ MeV). The results for the numerator ΔE_Q and the denominator ΔE_q are the same as used for plotting Fig. 2.

By rescaling the density of scattering centers with equation (2.2) [This case *ii*) is shown on the right hand side of Fig. 2.], we consider a class of model media in which the total number density of scattering centers is independent of m_t . This eliminates the first source of the strong m_t -dependence. So, these media have the same entropy density, irrespective of m_t , but they differ in their capacity to absorb recoil. At sufficiently high projectile momentum p , we see that the capacity to absorb recoil decreases with increasing target mass m_t . However, for sufficiently small momenta, when the mass of the projectile is not negligible for the scattering process, heavier projectiles can transfer their energy more efficiently to more massive scattering centers,

than to light ones. This leads to an inversion of the m_t -dependence of the average energy loss, which is particularly pronounced for the bottom quark (see lower right panel in Fig. 2).

One expects that for large projectile momentum, collisional energy loss depends negligibly on projectile mass. Consistent with this expectation, the curves for $\Delta E/E$ for fixed m_t but different projectile mass approach the same numerical value in the limit of large projectile momentum p . This is more clearly seen in Fig. 3, where the ratio $\Delta E_Q/\Delta E_q$ approaches unity for large projectile momentum p .

For finite projectile momentum p , Fig. 3 shows that a property of the medium (namely the value of m_t) determines whether the collisional energy loss of a massive partonic projectile is larger or smaller than that of a light projectile. This is in stark contrast to the case of radiative energy loss, where the dependence on projectile mass is expected to be governed entirely by the deadcone effect [13], which depends on properties of the projectile only. Some rough qualitative aspects of the inversion of the projectile mass hierarchy, seen in Fig. 3, may be understood on kinematical grounds. If the projectile is much heavier than the target scattering center, it has the tendency to simply 'run over' the target without significant change of momentum. In comparison, a light projectile of same momentum can transfer a larger fraction of its momentum to a target of similar mass. This implies that for a light target, the ratio $\Delta E_Q/\Delta E_q$ is smaller than unity. As the target mass is increased, the collisional energy loss of light projectiles always decreases, as seen in Fig. 2. However, if the total density of scattering centers is kept fixed, the energy loss of heavy quarks can increase with increasing m_t for the physics reasons explained in the context of Fig. 2 above. This implies that for sufficiently large m_t , $\Delta E_Q/\Delta E_q$ grows above unity: the projectile mass hierarchy is inverted.

5. Discussion

It has been pointed out recently that collisional parton energy loss can contribute to a medium-induced energy degradation of highly energetic partons in heavy ion collisions, which is comparable for heavy and for light quarks. This is in contrast to radiative energy loss mechanisms, which degrade the energy of massive projectiles less efficiently. The present study shows that, if suitable assumptions about the recoil properties of the medium are made, then the energy of heavy quarks may be quenched indeed as efficiently as that of light quarks. However, this finding depends strongly on the model-dependent properties of the medium. With relatively small changes of the model parameter m_t , one can also realize scenarios, in which light quarks are suppressed twice as much as bottom quarks for $p < 10$ GeV, or in which bottom quarks are suppressed more than light quarks. More generally, in the model scenarios explored here, small target masses are preferred if one wants to obtain a sizable contribution from collisional energy loss, but relatively large target masses are needed to obtain similar suppression factors for light and heavy quarks. A phenomenologically successful modeling of similar nuclear modification factors for light- and heavy-flavored hadrons in terms of collisional energy loss will need to satisfy these complementary constraints. We conclude that, if the relative strength of collisional energy loss can be determined independently, then the strong sensitivity of collisional energy

loss on m_t provides information about a property of the medium, which is distinct from its density.

We finally remark that determining a value of m_t in a model comparison to data does not imply a fortiori that the medium is a gas of quasi-particle of mass m_t . Rather, such an agreement could also be consistent with the picture of a strongly coupled medium, that does not carry quasi-particle excitations, but that absorbs recoil at a rate comparable to a gas of quasi-particles of mass m_t . In this latter case, which has received considerable support from RHIC data [1], the models explored here may still be able to parametrize with m_t the magnitude of longitudinal momentum transfer from the projectile to the medium, but they would be inadequate for a description of the strongly coupled dynamics of the medium.

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